Evaluation and Analysis of Thrust Units for Power-Limited Propulsion Systems

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Performance parameters of power-limited propulsion systems are expressed formally in terms of the kinematic properties of thrust units Ion engine deficiencies, such as neutral efflux, multiple ionization, and propellant stream divergence, are described as velocity distributions in the exhaust stream and are related algebraically to the important mission parameters The relative importance of these deficiencies is therefore determined in the framework of the mission The discussion provides a background for the selection of the best set of physical measurements to be made for the evaluation of thrust units and shows how these measurements are related to the mission parameters

Introduction

THERE are presently under development a number of propulsion systems which do not eject propellant at a single velocity The purpose of this study is to determine the extent to which a distribution in velocity of propellant ejection will degrade the propulsion efficiency, which is defined as a convenient mission parameter and determined by experimental measurements In general, the propellant is expelled from the thrust system as a diverging stream with some spread in speed, and the mass flow can be described by a velocity distribution function $\dot{M}(\bar{v})$, in terms of which several characteristics of the thrust system can be expressed For example, the total mass flow rate M, the thrust $\bar{T}$, and the kinetic power P in the propellant stream are given by the integrals

$$M = \int\int\int M(\bar{v})d\bar{v} \quad (1)$$

$$\bar{T} = \int\int\int M(\bar{v})\bar{v}d\bar{v} \quad (2)$$

$$P = \tfrac{1}{2}\int\int\int M(\bar{v})\bar{v}\ \bar{v}d\bar{v} \quad (3)$$

where $\bar{v}$ is the vector velocity coordinate and $d\bar{v}$ is the element of volume in velocity space By defining the averages,

$$\langle\bar{v}\rangle \equiv \int\int\int M(\bar{v})\bar{v}d\bar{v}/\int\int\int M(\bar{v})d\bar{v} \quad (4)$$

$$\langle\bar{v}\ \bar{v}\rangle \equiv \langle v^2\rangle = \int\int\int M(\bar{v})\bar{v}\ \bar{v}d\bar{v}/\int\int\int M(\bar{v})d\bar{v} \quad (5)$$

the thrust and propellant stream power can be written more familiarly as

$$\bar{T} = M\langle\bar{v}\rangle \quad (6)$$

$$P = \tfrac{1}{2}M\langle v^{23}\rangle \quad (7)$$

In discussing the relationships between mission parameters and thrust systems, it is helpful to examine the problem of rectilinear motion for a power-limited space vehicle starting from rest in free space, with the assumption that the characteristics of the propulsion system do not change during powered flight The basic equation of motion is

$$M(t)(du/dt) = M\langle v\rangle \quad (8)$$

where $M(t)$ is the instantaneous mass at time t, and u is the vehicle speed The scalar equation in u and $\langle v\rangle$ results from the assumption of rectilinear motion starting from rest The first integration of Eq (8) yields

$$u = \langle v\rangle \ln[M_0/M(t)] \quad (9)$$

where M_0 is the vehicle mass at zero time A second integration yields

$$x = \frac{M_0\langle v\rangle}{\dot{M}}\left[1 - \left(\frac{u}{\langle v\rangle} + 1\right)e^{-u/\langle v\rangle}\right] \quad (10)$$

where x is the displacement

System Optimization[1]

The total mass M_0 at zero time may be written as the sum of the propulsion system mass M_P, the propellant mass M_F, and the useful payload mass M_L:

$$M_0 = M_P + M_F + M_L \quad (11)$$

The mass at the end of the propulsion time τ may be written as

$$M_\tau = M_0 - M_F \quad (12)$$

where

$$M_F = M_\tau \quad (13)$$

From Eqs (7) and (13),

$$M_F = 2P\tau/\langle v^2\rangle \quad (14)$$

Following a practice common in the literature, we define the specific power α as the ratio of the electrical power P_E available at the output terminals of the power supply and the propulsion system mass M_P With this definition, the kinetic power P in the propellant stream is given by

$$P = \eta P_E = \eta\alpha M_P \quad (15)$$

and Eq (14) becomes

$$M_F = 2\eta\alpha\tau M_P/\langle v^2\rangle \quad (16)$$

Equation (15) defines an efficiency factor η for the conversion of input electrical power P_E to stream kinetic power P From Eqs (11) and (16),

$$M_P = (M_0 - M_L)(1 + 2\eta\alpha\tau/\langle v^2\rangle) \quad (17)$$

At the end of the propulsion period τ, Eq (9) may be written, by virtue of Eqs (11) and (12), in the form

$$M_F/M_0 = 1 - e^{-u/\langle v\rangle} \quad (18)$$

From Eqs (16) and (18),

$$M_P = (M_0\langle v^2\rangle/2\eta\alpha\tau)(1 - e^{-u/\langle v\rangle}) \quad (19)$$

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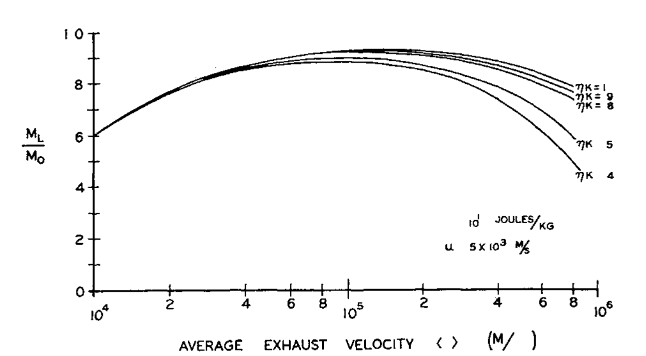


Fig 1 A graph of payload fraction vs average exhaust velocity for low vehicle velocity increments

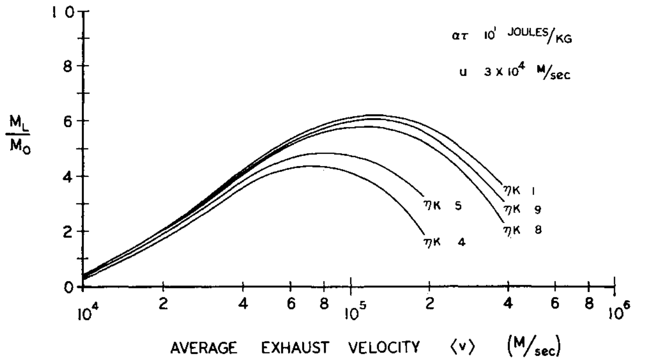


Fig 2 A graph of payload fraction vs average exhaust velocity for medium vehicle velocity increments

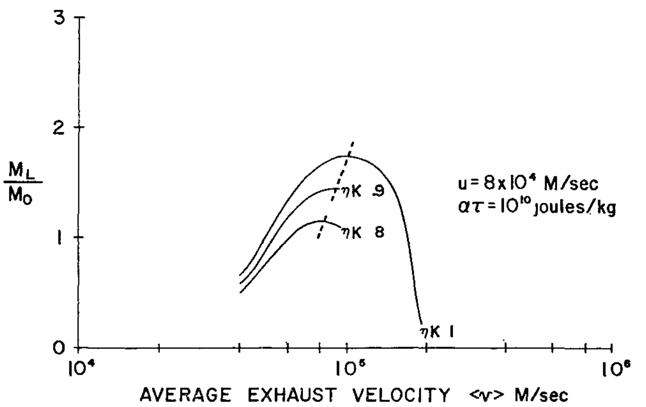


Fig 3 A graph of payload fraction vs average exhaust velocity for high vehicle velocity increments

Equating Eqs (17) and (19) yields

$$\frac{M_L}{M_0} = e^{-u/\langle v\rangle} - \frac{\langle v^2\rangle}{2\eta\alpha\tau}\,(1 - e^{-u/\langle v\rangle}) \tag{20}$$

The relationship between $\langle v\rangle$ and $\langle v^2\rangle$ in Eq (20) is obviously determined by the propellant velocity distribution function $\dot{M}(\bar{v})$, according to Eqs (4) and (5) A parameter k is defined as

$$k \equiv \langle v\rangle^2/\langle v^2\rangle \tag{21}$$

so that Eq (20) becomes

$$\frac{M_L}{M_0} = e^{-u/\langle v\rangle} - \frac{\langle v\rangle^2}{2k\eta\alpha\tau}\,(1 - e^{-u/\langle v\rangle}) \tag{22}$$

Figures 1–3 show graphs of M_L/M_0 as a function of $\langle v\rangle$ for various values of the parameters u, η, α, and τ A plot of optimum $\langle v\rangle$ vs ηk for the conditions of Fig 2 is presented in Fig 4 By choosing the optimum $\langle v\rangle$ for a given ηk from Fig 4 and substituting this $\langle v\rangle$ into Eqs (14) and (16), the data for M_F/M_0 and M_P/M_0, presented in Fig 5, were obtained In Figs 6–8, Eq (22) has been solved for u, which has been plotted as a function of $\langle v\rangle$ for various sets of parameters In Fig 9, the payload fraction is plotted as a function of ηk, with the terminal velocity u as a parameter

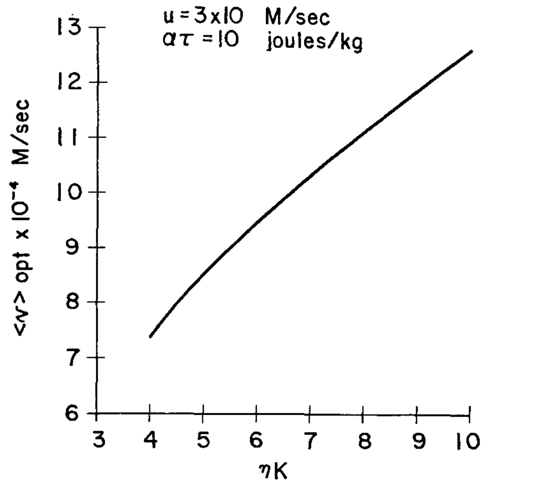


Fig 4 A graph of optimum average velocity of propellant ejection vs the beam efficiency factor for medium vehicle velocity increments

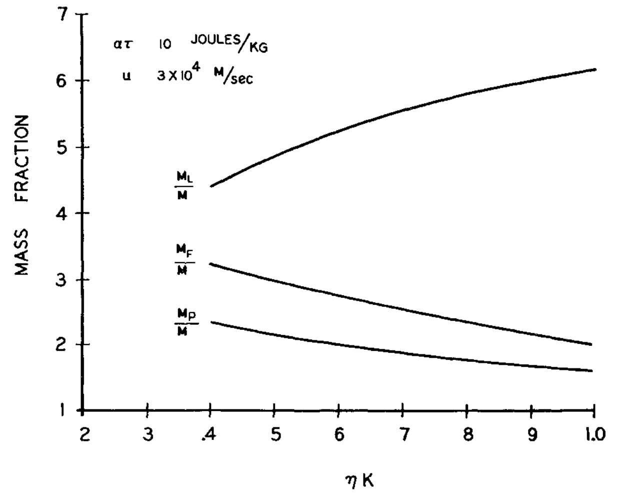


Fig 5 Mass fractions for maximum M_L/M_0 as a function of beam efficiency factor

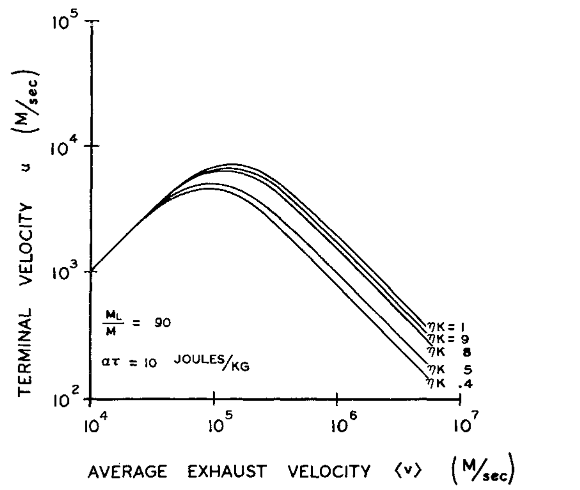
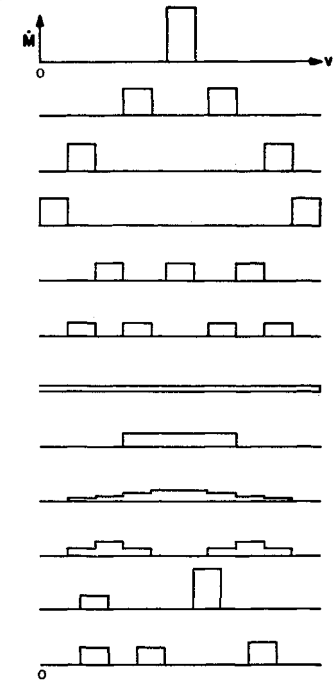


Fig 6 A plot of terminal velocity vs exhaust velocity for large payload mass fraction

It can be shown that k lies between zero and unity for any arbitrary distribution in propellant velocity Since k is determined completely by the propellant stream characteristics and has the form of an efficiency factor, it will be

Table 1 Paiameter k for various distribution functions, $M(v)$ where $k = \langle v\rangle^2/\langle v^2\rangle$



k =
1 00
0 92
0 67
0 55
0 85
0 77
0 75
0 95
0 89
0 79
0 89
0 79

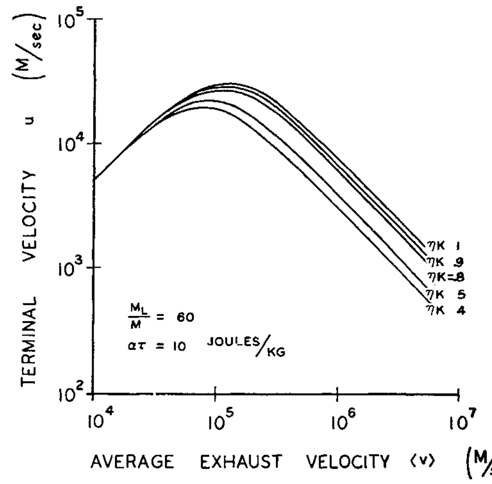


Fig 7 A plot of terminal velocity vs aveiage exhaust velocity with intermediate payload mass fraction

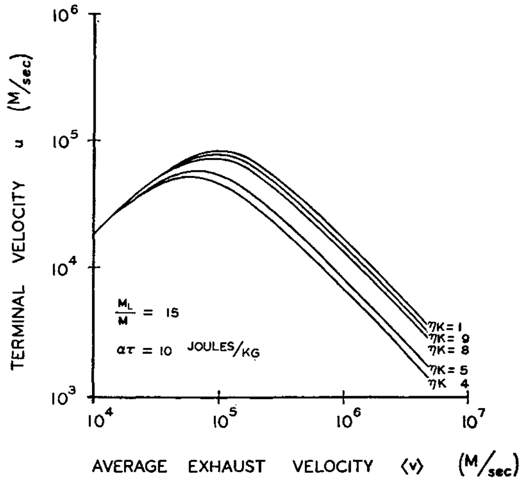


Fig. 8 A plot of terminal velocity vs average exhaust velocity for small payload mass fraction

called the beam power efficiency in agreement with Hunter [2] By forming the product ηk from Eqs (6, 7, 15, and 21), it is seen that

$$\eta k = T^2/2P_E M \tag{23}$$

It is possible to show that ηk expressed in this form follows naturally from the comparison of an ideal and a realistic thrust unit using the same input power P_E and the same mass flow rate M Hunter[2] has obtained the expression for k, as given in Eq (21), by fixing the specific impulse (or $\langle v\rangle$) and comparing ideal and actual thrust units

It is important to note that the product ηk is readily established in terms of the experimental observables or the right-hand side of Eq (23) and that η and k cannot be determined separately except by extensive physical measurements It might be argued, therefore, that Eq (23) provides a logical basis for the evaluation of thrust units However, the maximization of M_L/M_0 from Eq (22) is with respect to $\langle v\rangle$, and, since $k\eta\alpha\tau$ is a function of $\langle v\rangle$, system optimization becomes much more complicated For that reason, it would be practical to prepare a computer program to compute M_L/M_0 from Eq (22), with values of ηk and $\langle v\rangle$ obtained by the substitution of measured values of P_E, M, and T into Eqs (23) and (6), and to choose the $\langle v\rangle$ for which M_L/M_0 is maximum Because of radiation shielding and power conditioning requirements, even α and τ will show some dependence on $\langle v\rangle$

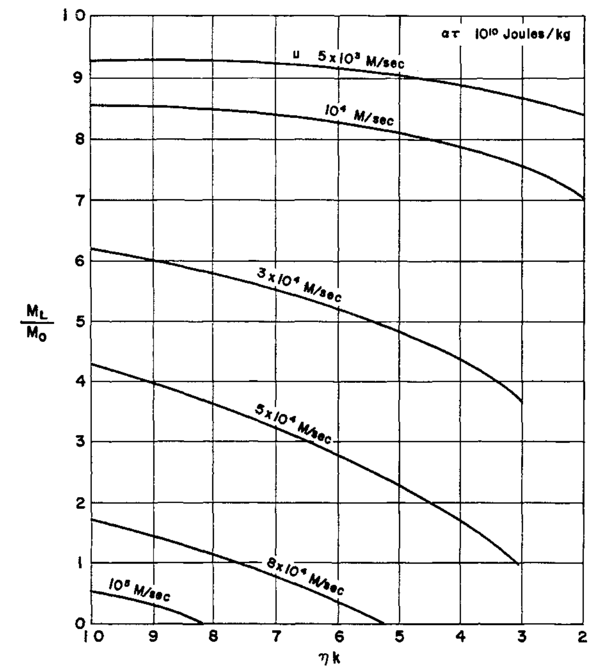


Fig 9 Payload plotted as a function of k with the terminal velocity u as parameter

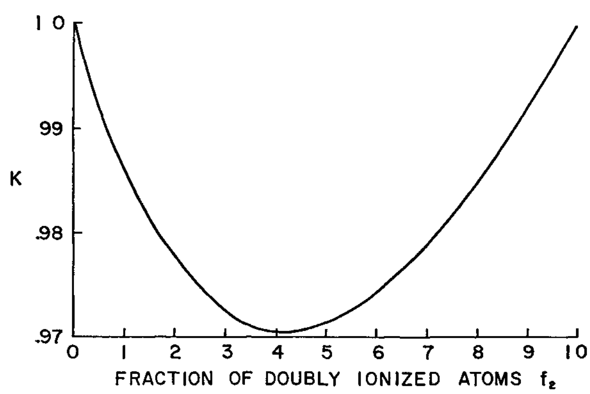


Fig 10 K as a function of double ionization for an ion gun emitting singly and doubly ionized atoms

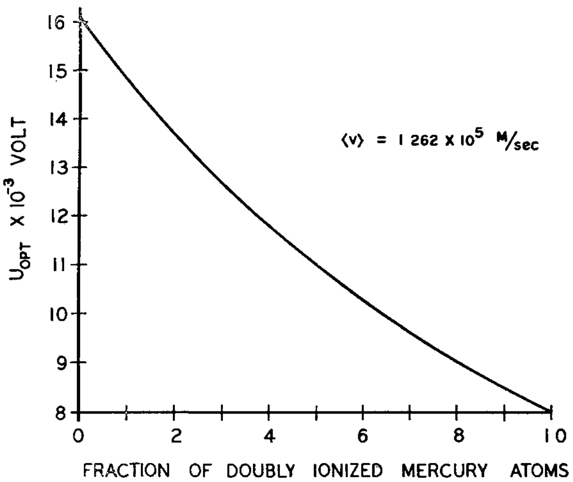


Fig 11 The required acceleration potential as a function of the doubly ionized fraction of Hg

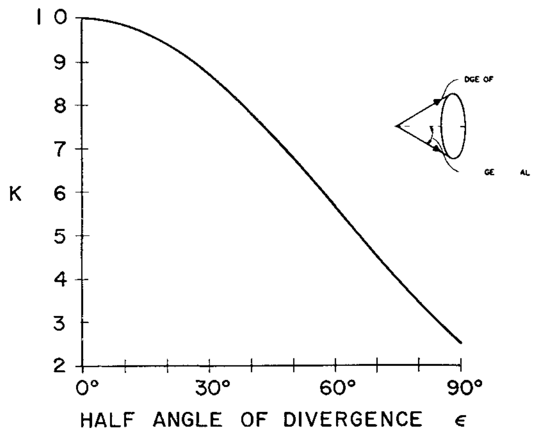


Fig 12 The dependence of *K* on the divergence half-angle

Beam Power Efficiencies for Various Propulsion Concepts

Although the beam power efficiency is not easily determined experimentally, it can be computed readily from Eqs (4, 5, and 21) for a variety of simple distribution functions Table 1 gives the value of k for several simple distributions in speed with no spread in directions In these cases, the integrals of Eqs (4) and (5) are replaced by sums, and k takes the form

$$k \equiv \langle v\rangle^2/\langle v^2\rangle = (\Sigma_i f_i v_i)^2/\Sigma_i f_i v_i^2 \qquad (24)$$

where f_i is the fraction of the mass which is expelled with speed v_i

For the case of multiple ionization, the ion velocity v_n is given by

$$v_n = (2neV/m)^{1/2} \qquad (25)$$

where n is the number of electrons removed, e is the electron charge, V is the accelerating potential, and m is the ion mass Combining Eqs (24) and (25), we have

$$k = (\Sigma f_n n^{1/2})^2/\Sigma_n f_n \qquad (26)$$

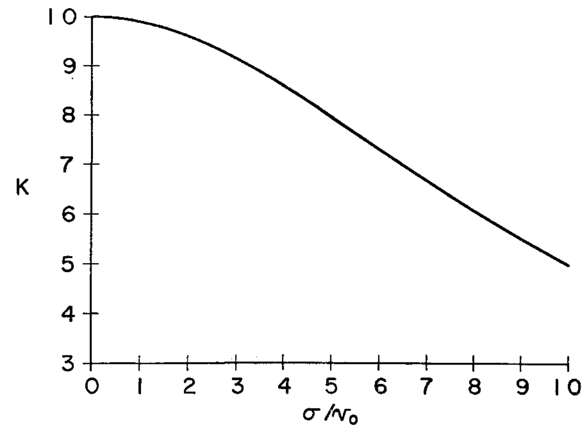


Fig 13 *K* vs standard deviation average velocity ratio

For the case of neutral efflux and singly ionized atoms, k is seen from Eq (26) to be equal to the fraction of ionized atoms For singly and doubly ionized atoms, Eq (26) yields

$$k = [1 + (2^{1/2} - 1)f_2]^2/(1 + f_2) \qquad (27)$$

where f_2 is the fraction of doubly ionized atoms Equation (27) is plotted in Fig 10 The optimum accelerating potential is given by

$$U = (M\langle v\rangle^2/2e)/(\Sigma n^{1/2} f_n)^2 \qquad (28)$$

where it is assumed that the optimum $\langle v\rangle$ and the different ion fractions are known A graph of U for a stream composed of various mixtures of singly and doubly ionized atoms is shown in Fig 11

A particularly simple example of the effect of beam divergence on k is obtained by assuming a single speed v with an isotropic directional distribution up to some limiting half-angle ϵ In this case

$$k = (1 + \cos\epsilon)^2/4 \qquad (29)$$

and the result is plotted in Fig 12

If the mass distribution has the form of an error function, the beam power efficiency can be written as

$$k = 1/[1 + (\sigma/v_0)^2] \qquad (30)$$

where v_0 is the average speed, and σ is the standard deviation Figure 13 shows a plot of k vs σ/v_0

Summary

The optimization of power-limited propulsion systems has been examined from the viewpoint of various imperfections in thrust units Particular attention has been paid to the penalties associated with various velocity distributions in the propellant stream, and the problem of evaluating thrust units has been examined from the systems viewpoint

References

[1] Stuhlinger, E and Seitz, R N , "Electrostatic propulsion systems for space vehicles," *Advances in Space Science* (Academic Press Inc , New York, 1960), Vol II, pp 264–350

[2] Hunter, R E , "Theoretical consideration of nonuniformly charged expellant beams,' Aeronaut Res Lab TN 60 138 (1960)